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Channel Estimation in OFDM Systems with Unknown Interference

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INTRODUCTION

- The Use of OFDM in an interfered scenario poses several technical challenges. Channel estimation (CE) is of primary importance.
- CE approaches: preamble and pilot
- These schemes provide good results as long as received signal is only affected by MP and thermal noise.
- NBI? Available methods require knowledge of the NBI position. Some methods estimate interference (**I**) power at pilot positions. NBI with unknown power?
- CE with unknown **I**. Use of training blocks or pilot symbols and estimate CIR by treating the **I** power on each pilot subcarrier as a nuisance parameter.
- ML with EM/Jacobi-Newton algorithm.

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Signal Model and Problem Formulation

CIR:

$$\mathbf{h} = [h(1), h(2), \ldots, h(L)]^T$$

DFT of kth rec blck:

$$X(n,k) = c(n,k)H(n) + w(n,k) \quad -N_\alpha \leq n \leq N_\alpha \quad (1)$$

CFR:

$$H(n) = \sum_{\ell=1}^{L} h(\ell) e^{-j2\pi n(\ell-1)/N} \qquad -N_\alpha \leq n \leq N_\alpha. \ (2)$$

$\{a(p,k); 1 \leq p \leq P\}$ with constant energy $\sigma_s^2 = |a(p,k)|^2$

set $J_p = \{n_p; 1 \leq p \leq P\}$

DFT O/P at the pilot positions:

$$\mathbf{X}(k) = [X(n_1,k), X(n_2,k), \ldots, X(n_P,k)]^T$$

From (1) and (2):

$$\mathbf{X}(k) = \mathbf{A}(k)\mathbf{F}\mathbf{h} + \mathbf{w}(k) \qquad (3)$$

SIGNAL MODEL AND PROBLEM FORMULATION…

$$\mathbf{X}(k) = \mathbf{A}(k)\mathbf{F}\mathbf{h} + \mathbf{w}(k) \qquad (3)$$

$$\mathbf{A}(k) = \mathrm{diag}\{a(p,k); 1 \leq p \leq P\}$$

$$[\mathbf{F}]_{p,\ell} = e^{-j2\pi n_p(\ell-1)/N} \qquad 1 \leq p \leq P, \quad 1 \leq \ell \leq L \quad (4)$$

$$\mathbf{w}(k) = [w(n_1,k), w(n_2,k), \ldots, w(n_P,k)]^T$$

Entries are assumed to be Gaussian distributed with zero mean and unknown variance:

$$\sigma^2(n_p) = \sigma_w^2 + \sigma_I^2(n_p)$$

Also, Statistically independent

Thermal noise contribution

Average NBI power, which is assumed constant over the observation period

SIGNAL MODEL AND PROBLEM FORMULATION…

$$\mathbf{w}(k) = [w(n_1, k), w(n_2, k), \ldots, w(n_P, k)]^T$$

Gaussian vector → Zero mean

↓

Diagonal covariance matrix

$$\mathbf{C} = \mathrm{diag}\{\sigma^2(n_p); 1 \leq p \leq P\}$$

Estimation Problem:

$\{\mathbf{H}, \boldsymbol{\sigma}^2\}$ → Set of unknown parameters

$$\boldsymbol{\sigma}^2 = [\sigma^2(n_1), \sigma^2(n_2), \ldots, \sigma^2(n_P)]^T$$

(2) In matrix notation:

$$\mathbf{H} = \mathbf{G}\mathbf{h} \qquad (5)$$

$$[\mathbf{G}]_{n,\ell} = e^{-j2\pi(n-N_\alpha-1)(\ell-1)/N} \quad 1 \leq n \leq N_u, \quad 1 \leq \ell \leq L. \qquad (6)$$

Signal Model and Problem Formulation…

Estimated CFR: $\hat{\mathbf{H}} = \mathbf{G}\hat{\mathbf{h}}$ (7)

Reducing the unknown parameters! Nu>>L

Observations for ML approach:

$$\mathbf{X} = [\mathbf{X}^T(1), \mathbf{X}^T(2), \ldots, \mathbf{X}^T(K)]^T$$

K adjacent blocks

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ML Channel Estimation

Given the unknown parameters $(\mathbf{h}, \boldsymbol{\sigma}^2)$

Vectors $\{\mathbf{X}(k)\}$in (3) are statistically independent and Gaussian distributed with mean $\mathbf{A}(k)\mathbf{F}\mathbf{h}$ and covariance matrix $\mathbf{C}$.

Joint PDF:

$$p(\mathbf{X} \,|\, \tilde{\mathrm{h}}, \tilde{\sigma}^2) = \prod_{p=1}^{P} \frac{1}{[\pi\tilde{\sigma}^2(n_p)]^K} \exp\left\{-\frac{1}{\tilde{\sigma}^2(n_p)} \sum_{k=1}^{K} \left|Z_k(p, \tilde{\mathrm{h}})\right|^2\right\} \quad (8)$$

where

$$Z_k(p, \tilde{\mathrm{h}}) = X(n_p, k) - a(p, k) \sum_{\ell=1}^{L} \tilde{h}(\ell) e^{-j2\pi n_p(\ell-1)/N}. \quad (9)$$

ML Channel Estimation…

LLF:

$$\Lambda(\tilde{\mathrm{h}}, \tilde{\sigma}^2) = -K \sum_{p=1}^{P} \ln[\pi \tilde{\sigma}^2(n_p)] - \sum_{k=1}^{K} \sum_{p=1}^{P} \frac{1}{\tilde{\sigma}^2(n_p)} \left| Z_k(p, \tilde{\mathrm{h}}) \right|^2 \qquad (10)$$

Maximizing $\Lambda(\tilde{\mathrm{h}}, \tilde{\sigma}^2)$ with respect to $\tilde{\sigma}^2(n_p)$ yields

$$\hat{\sigma}^2(n_p; \tilde{\mathrm{h}}) = \frac{1}{K} \sum_{k=1}^{K} \left| Z_k(p, \tilde{\mathrm{h}}) \right|^2 \qquad 1 \leq p \leq P. \qquad (11)$$

Concentrated LF:

$$\Gamma(\tilde{\mathrm{h}}) = -\sum_{p=1}^{P} \ln \left[\sum_{k=1}^{K} \left| Z_k(p, \tilde{\mathrm{h}}) \right|^2 \right]. \qquad (12)$$

Maximum → ML estimate of CIR

??

ML CHANNEL ESTIMATION…

Alternate approach: Consider σ^2 as a nuisance random vector which is averaged out from (8) to yeild the marginal LF of **h**. a-priori pdf for σ^2 required.

Inverse-Gamma pdf

Design parameter

$$p(\sigma^2) = \frac{\lambda}{\sigma^4} \exp\left\{-\frac{\lambda}{\sigma^2}\right\}, \qquad \text{for} \quad \sigma^2 > 0 \qquad (14)$$

Marginal LF:

$$p(\mathbf{X} \,|\, \tilde{\mathbf{h}}) = \prod_{p=1}^{P} \int_0^\infty \frac{\lambda}{\sigma^4 (\pi\sigma^2)^K} \exp\left\{-\frac{1}{\sigma^2}\left[\lambda + \sum_{k=1}^{K} \left|Z_k(p,\tilde{\mathbf{h}})\right|^2\right]\right\} d\sigma^2 \qquad (15)$$

ML Channel Estimation…

$$p(\mathbf{X}\,|\tilde{\mathrm{h}}) = \prod_{p=1}^{P} \frac{\lambda}{\pi^K} \frac{K!}{\left[\lambda + \sum_{k=1}^{K} \left|Z_k(p,\tilde{\mathrm{h}})\right|^2\right]^{K+1}} \quad (16)$$

after letting $\sigma^2 = 1/t$ and using the identity

$$\int_0^\infty t^L e^{-\alpha t} dt = \frac{L!}{\alpha^{L+1}}. \quad (17)$$

Marginal LLF:

$$\ln[p(\mathbf{X}\,|\tilde{\mathrm{h}})] = P \ln\left(\lambda \frac{K!}{\pi^K}\right) - (K+1)\sum_{p=1}^{P} \ln\left[\lambda + \sum_{k=1}^{K} \left|Z_k(p,\tilde{\mathrm{h}})\right|^2\right]. \quad (18)$$

ML Channel Estimation…

(12)

Objective Function:

$$\Phi(\tilde{\mathbf{h}}) = -\sum_{p=1}^{P} \ln\left[\lambda + \sum_{k=1}^{K} \left|Z_k(p, \tilde{\mathbf{h}})\right|^2\right] \qquad (19)$$

MLE of **h:**

$$\hat{\mathbf{h}}_{MLE} = \arg\max_{\tilde{\mathbf{h}}} \{\Phi(\tilde{\mathbf{h}})\} \qquad (20)$$

Asymptotic efficiency property of MLE

Unbiased for large data records

Covariance matrix attains $(\text{FIM})^{-1}$

FIM in closed-form from (19) is intractable, found from (10)

ML Channel Estimation…

$$\mathbf{E}\{(\hat{\mathbf{h}}-\mathbf{h})(\hat{\mathbf{h}}-\mathbf{h})^H\} - \frac{1}{K\sigma_s^2}\cdot(\mathbf{F}^H\mathbf{C}^{-1}\mathbf{F})^{-1} \geq 0 \quad (21)$$

Using (7) and (21),

$$\mathbf{E}\{(\hat{\mathbf{H}}_{MLE}-\mathbf{H})(\hat{\mathbf{H}}_{MLE}-\mathbf{H})^H\} - \frac{1}{K\sigma_s^2}\cdot\mathbf{G}(\mathbf{F}^H\mathbf{C}^{-1}\mathbf{F})^{-1}\mathbf{G}^H \geq 0 \quad (22)$$

$$\mathbf{E}\left\{\left\|\hat{\mathbf{H}}_{MLE}-\mathbf{H}\right\|^2\right\} \geq \frac{1}{K\sigma_s^2}\cdot\mathrm{tr}\{\mathbf{G}(\mathbf{F}^H\mathbf{C}^{-1}\mathbf{F})^{-1}\mathbf{G}^H\}. \quad (23)$$

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ITERATIVE CHANNEL ESTIMATION

EM algorithm

Complete data set: $(\mathbf{X}, \sigma^2)$ **Unknown parameter:** $\mathbf{h}$

E-step

$$Q\left(\tilde{\mathbf{h}} \middle| \hat{\mathbf{h}}_i\right) = \mathbf{E}_{\sigma^2}\left\{\ln\left[p\left(\mathbf{X} \middle| \sigma^2, \tilde{\mathbf{h}}\right)\right] p\left(\mathbf{X} \middle| \sigma^2, \hat{\mathbf{h}}_i\right)\right\} \quad (24)$$

M-step

$$\hat{\mathbf{h}}_{i+1} = \arg\max_{\tilde{\mathbf{h}}}\left\{Q\left(\tilde{\mathbf{h}} \middle| \hat{\mathbf{h}}_i\right)\right\}. \quad (25)$$

$$\Psi\left(\tilde{\mathbf{h}} \middle| \hat{\mathbf{h}}_i\right) = -\frac{1}{K}\sum_{k=1}^{K}[\mathbf{X}(k) - \mathbf{A}(k)\mathbf{F}\tilde{\mathbf{h}}]^H \hat{\mathbf{C}}_i^{-1}[\mathbf{X}(k) - \mathbf{A}(k)\mathbf{F}\tilde{\mathbf{h}}] \quad (26)$$

where

$$\hat{\mathbf{C}}_i = \mathrm{diag}\{\hat{\sigma}_i^2(n_p); 1 \le p \le P\} \quad (27)$$

and

$$\hat{\sigma}_i^2(n_p) = \frac{\lambda}{K} + \frac{1}{K}\sum_{k=1}^{K}\left|Z_k(p, \hat{\mathbf{h}}_i)\right|^2 \quad 1 \le p \le P \quad (28)$$

March 23, 2011 TelLab, Inha Univ, Korea

Iterative Channel Estimation…

CIR is the max of (26) with respect to $\tilde{\mathbf{h}}$.

$$\hat{\mathbf{h}}_{i+1} = (\mathbf{F}^H \hat{\mathbf{C}}_i^{-1} \mathbf{F})^{-1} \mathbf{F}^H \hat{\mathbf{C}}_i^{-1} \mathbf{Y} \qquad (29)$$

with

$$\mathbf{Y} = \frac{1}{K\sigma_s^2} \sum_{k=1}^{K} \mathbf{A}^H(k)\mathbf{X}(k). \qquad (30)$$

Neglect NBI for initial guess:

$$\hat{\mathbf{h}}_1 = (\mathbf{F}^H \mathbf{F})^{-1} \mathbf{F}^H \mathbf{Y}. \qquad (31)$$

(28) & (29) EMCE

CFR (from (7)):

$$\hat{\mathbf{H}}_{EMCE} = \mathbf{G}\hat{\mathbf{h}}_J \qquad (32)$$

If J=1, MLE-IFS:

$$\hat{\mathbf{H}}_{MLE-IFS} = \mathbf{G}(\mathbf{F}^H \mathbf{F})^{-1} \mathbf{F}^H \mathbf{Y}. \qquad (33)$$

Iterative Channel Estimation…

Jacobi-Newton algorithm:

One possible drawback of EMCE

Inverting of $\mathbf{F}^H \hat{\mathbf{C}}_i^{-1} \mathbf{F}$ at each new iteration

$$\hat{\mathbf{h}}_{i+1} = \hat{\mathbf{h}}_i - \mathbf{D}_i^{-1} \nabla \Phi(\tilde{\mathbf{h}}) \Big|_{\hat{\mathbf{h}}_i} \quad (34)$$

$$\mathbf{D}_i = \mathrm{diag}\{\mu_i(\ell); \ell = 1, 2, \ldots, L\}$$

$$\mu_i(\ell) = \left[\frac{\partial^2 \Phi(\tilde{\mathbf{h}})}{\partial \tilde{h}^2(\ell)} \right]_{\tilde{\mathbf{h}} = \hat{\mathbf{h}}_i}. \quad (35)$$

$$\hat{\mathbf{h}}_{i+1} = \hat{\mathbf{h}}_i + \frac{1}{\mathrm{tr}\{\hat{\mathbf{C}}_i^{-1}\}} \mathbf{F}^H \hat{\mathbf{C}}_i^{-1} (\mathbf{Y} - \mathbf{F}\hat{\mathbf{h}}_i) \quad (39)$$

$$\hat{\mathbf{H}}_{JNCE} = \mathbf{G}\hat{\mathbf{h}}_J. \quad (40)$$

Complexity:

	Number of flops
EMCE	$P(4L + 6K - 1 + 10\log_2 P + 16L^2)$
JNCE	$P(9 + 6K + 10\log_2 P)$

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- Introduction
- Signal Model and Problem Formulation
- Maximum-Likelihood (ML) Channel Estimation
- Iterative Channel Estimation
- **Results and Discussion**

Results and Discussion

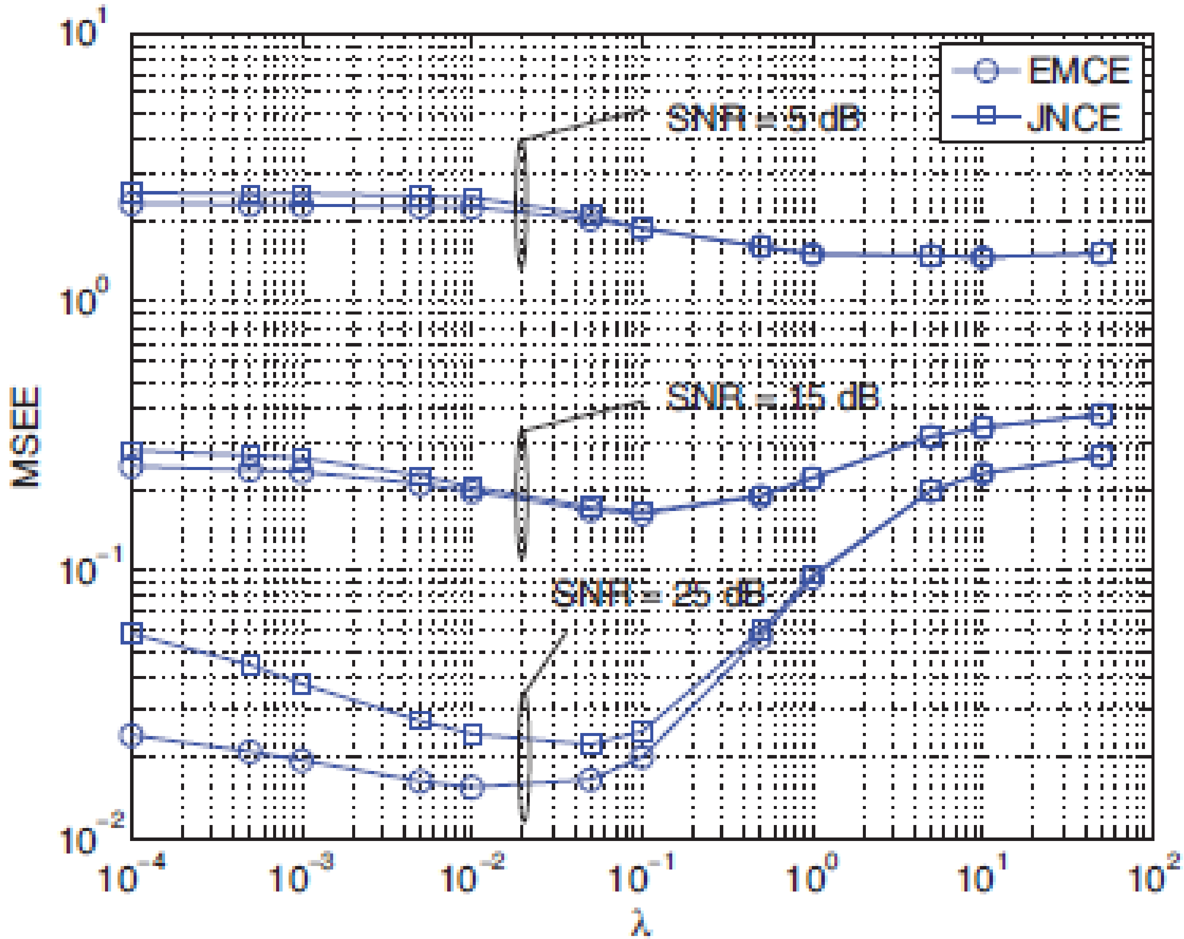

Fig. 1. Performance of the proposed channel estimators as a function of λ for SIR = 0 dB.

Results and Discussion…

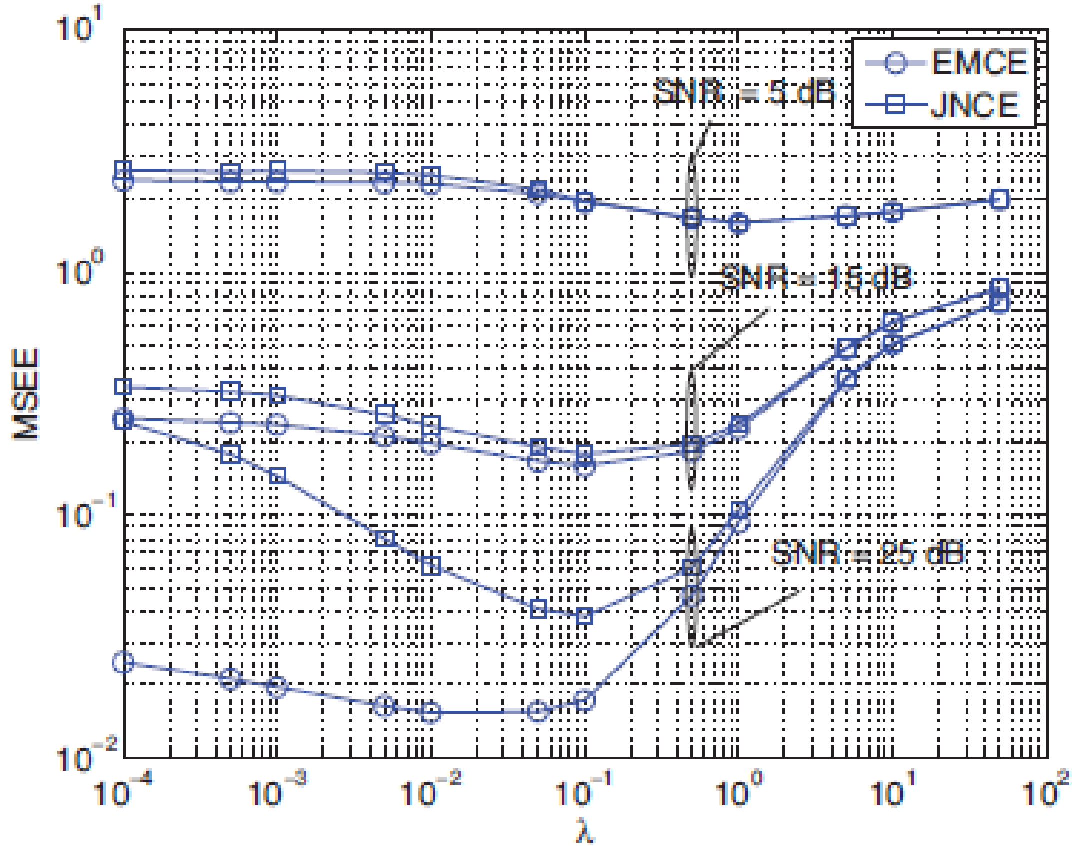

Fig. 2. Performance of the proposed channel estimators as a function of λ for SIR = −5 dB.

RESULTS AND DISCUSSION…

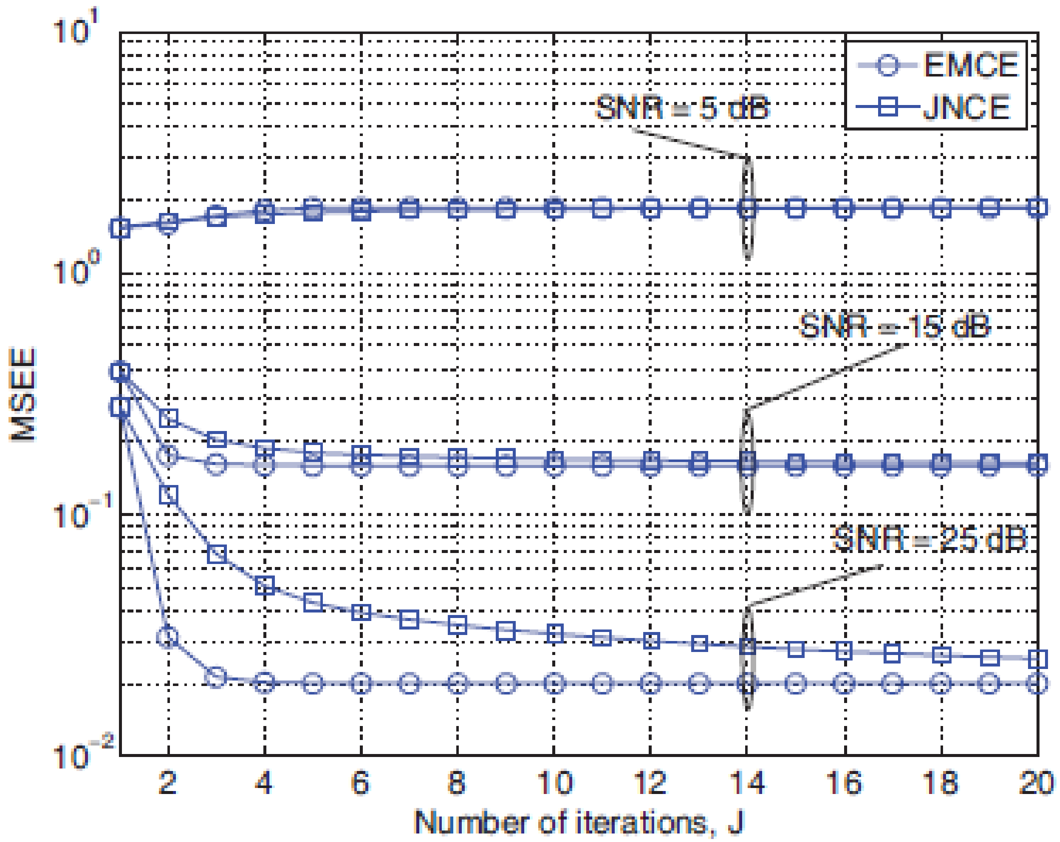

Fig. 3. Performance of the proposed estimators as a function of J for SIR $= 0$ dB

Results and Discussion…

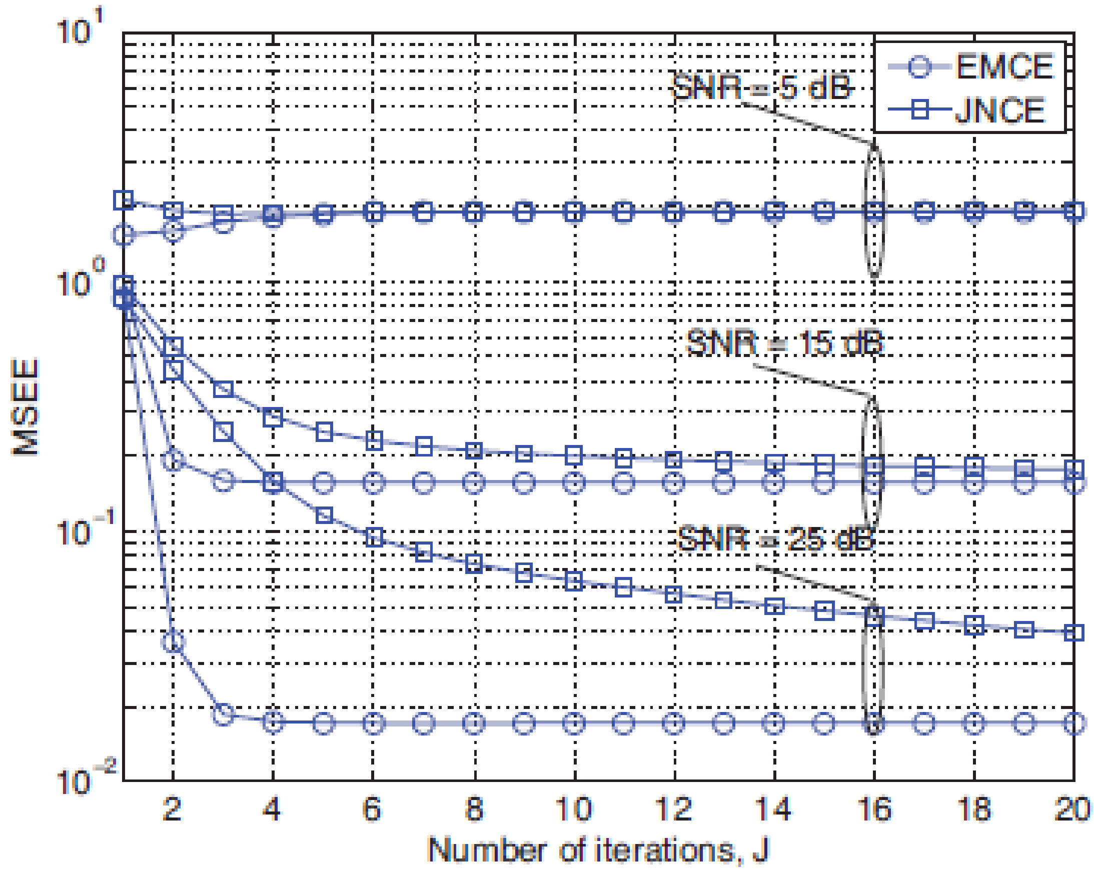

Fig. 4. Performance of the proposed estimators as a function of J for SIR $= -5$ dB

Results and Discussion…

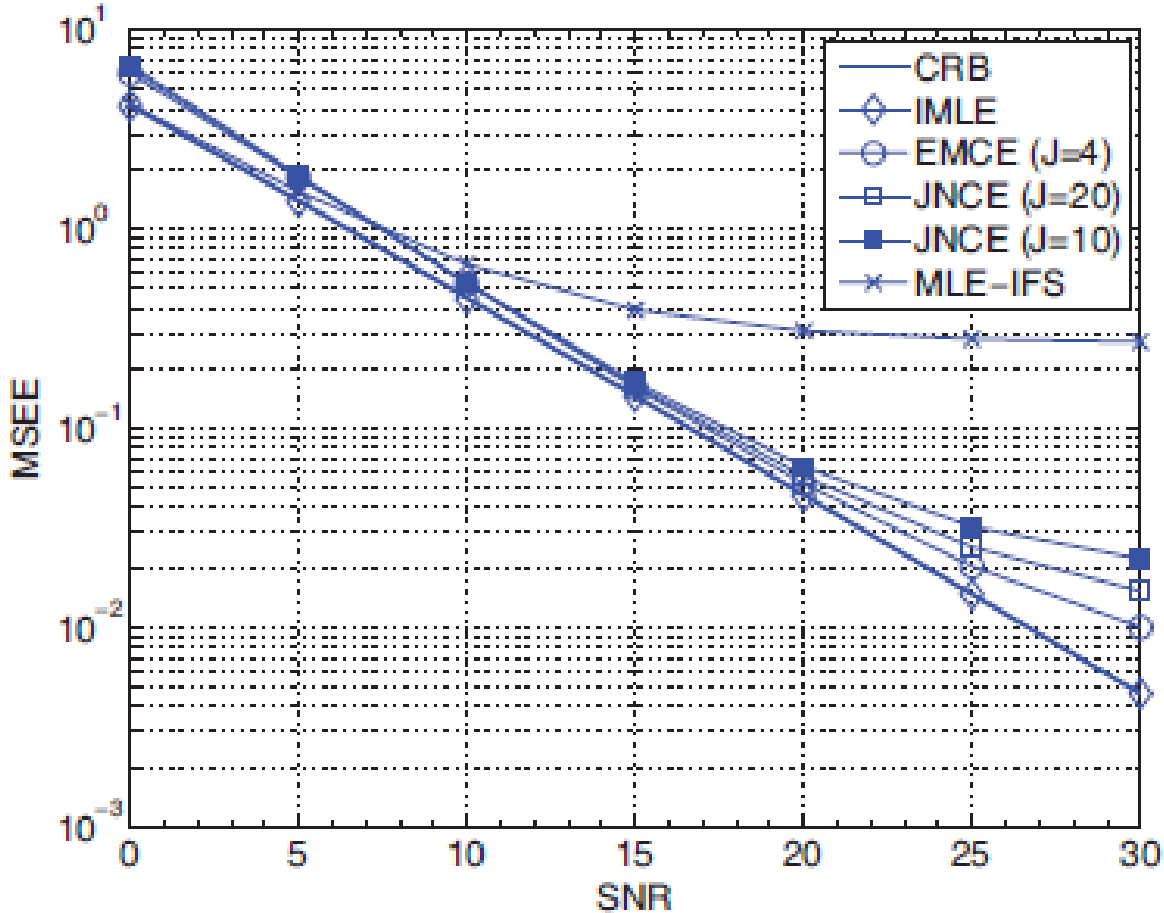

Fig. 5. Performance of the proposed estimators as a function of the SNR for SIR = 0 dB

Results and Discussion…

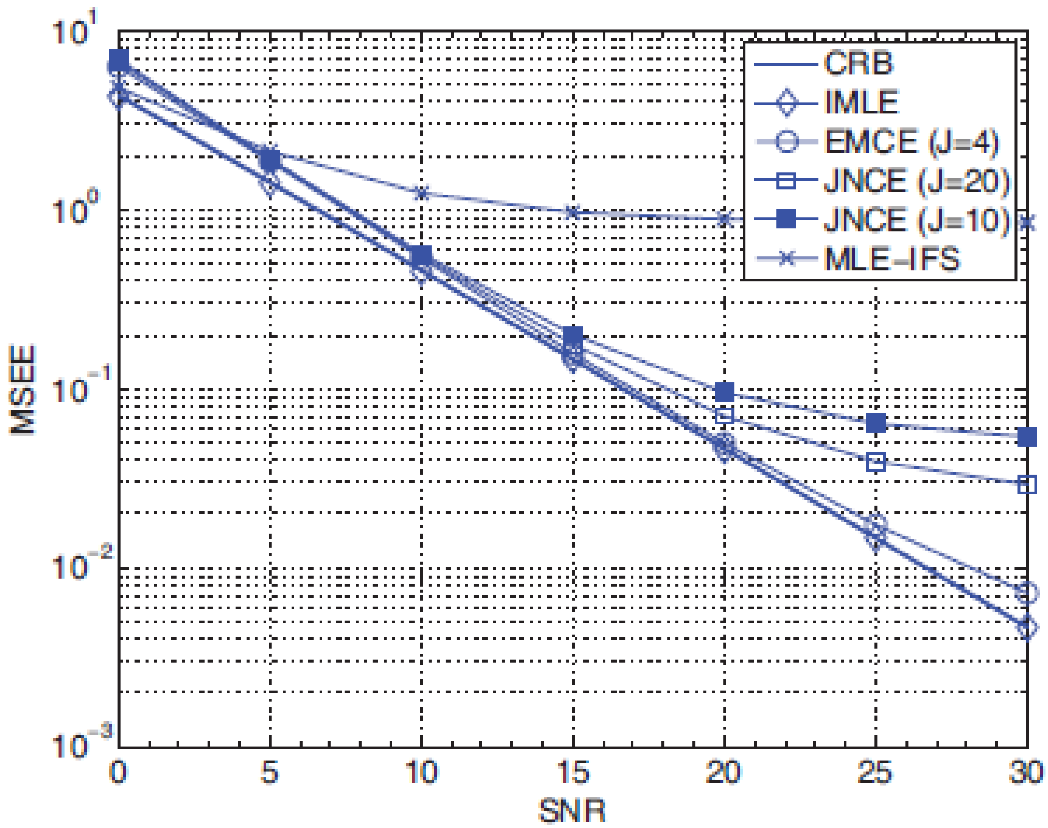

Fig. 6. Performance of the proposed estimators as a function of the SNR for SIR = −5 dB.

Results and Discussion…

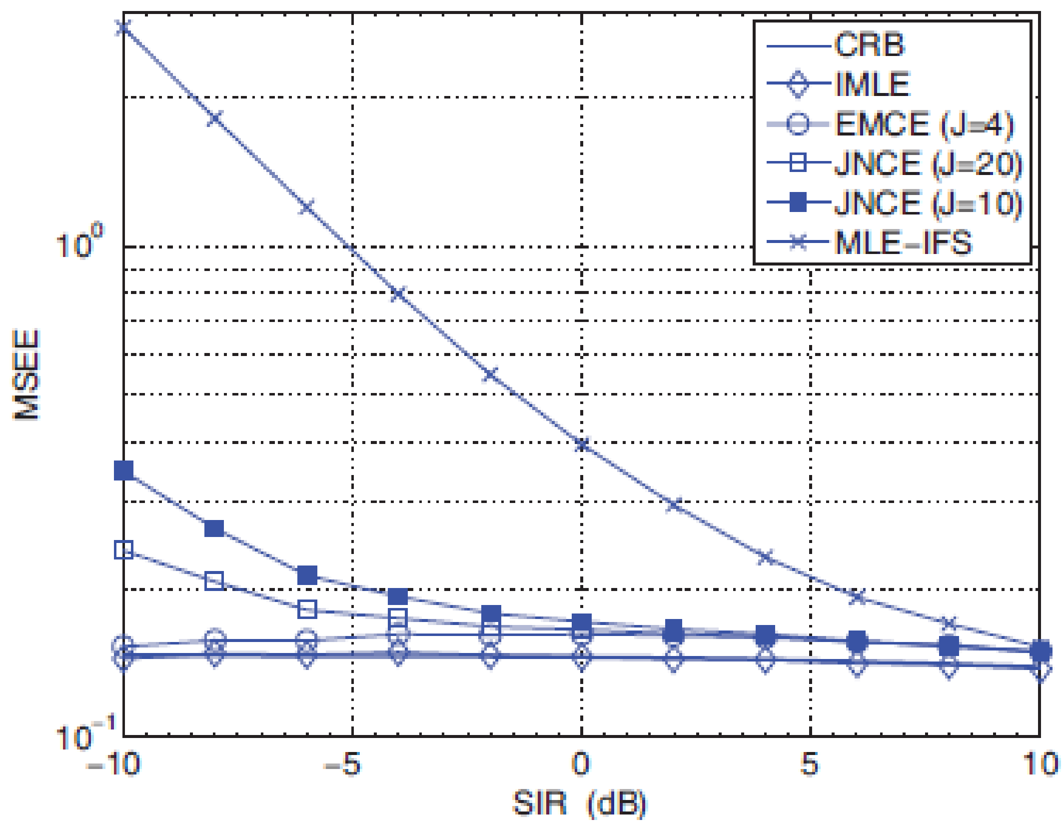

Fig. 7. Performance of the proposed estimators as a function of the SIR for SNR = 15 dB.

RESULTS AND DISCUSSION…

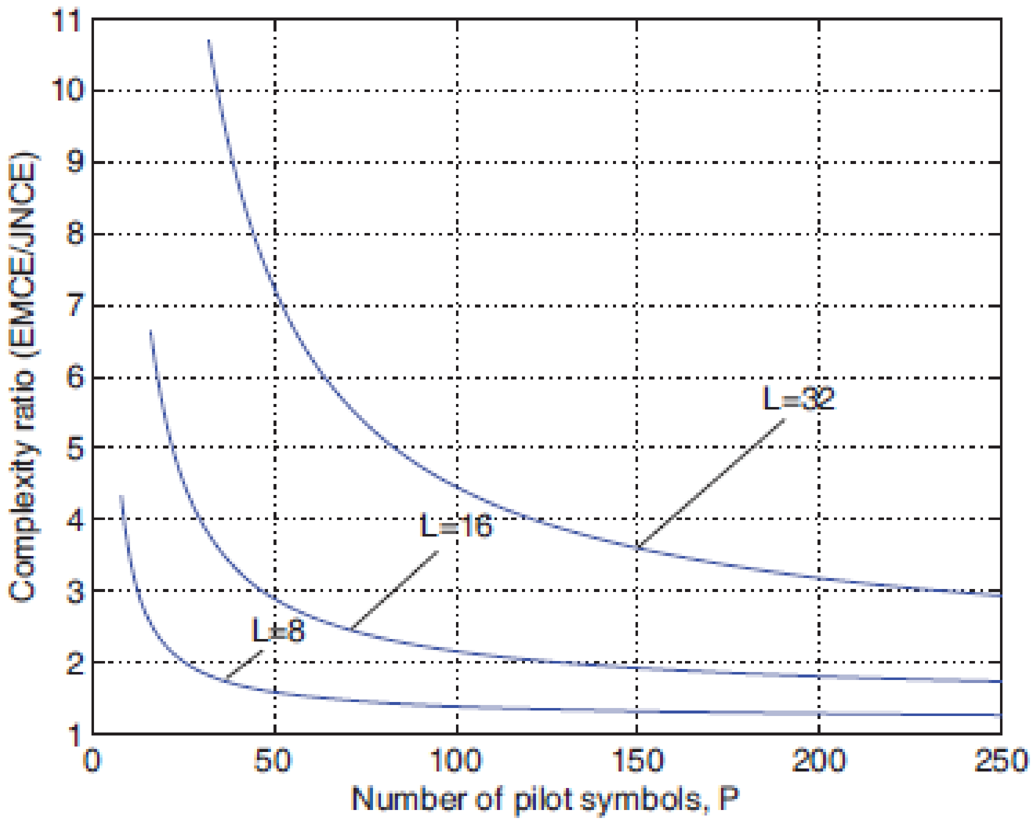

Fig. 8. Complexity comparison between EMCE and JNCE as a function of P.

Discussion

- Two schemes for CE in OFDM-based systems in presence of NBI
- NBI is modeled as a Gaussian process and its power is averaged out from the LF function using inverse-gamma distribution.
- Iterative solution: EM and JN.
- Proposed methods are inherently robust to NBI and can effectively used in a severely interfered scenario.
- ECMA or JNCE? Application-centric.

Thanks: Questions or Comments